Algebraic Groups and Geometrization of the Langlands Program
(ENS-Lyon/Université Lyon 1)

Conference Programme, 22-25 May

All talks will take place at *École Normale Suprieure de Lyon*. Directions to get there can be found here:

`http://www.umpa.ens-lyon.fr/umpa/contact-et-acces`

Tuesday, 22 May

9h15-10h — Welcome
10h- 11h — Roman Fedorov
11h30-12h30 — Najmuddin Fakhruddin

Lunch

14h-15h — Igor Rapinchuk
15h30-16h30 — Kęstutis Česnavičius

Wednesday, 23 May

10h- 11h — George McNinch
11h30-12h30 — Michael Harris

Lunch

14h-15h — Javier Fresán
15h30-16h30 — Nicolas Templier

Thursday, 24 May

10h- 11h — Jared Weinstein
11h30-12h30 — Arthur-César Le Bras

Lunch

14h-15h — Tasho Kaletha
15h30-16h30 — Alexander Smith

Friday, 25 May

9h30- 10h30 — Vladimir Chernousov
10h45-11h45 — Fei Xu

Lunch

13h15-14h15 — Sophie Morel
14h30-15h30 — Joseph Ayoub

Abstracts

Joseph Ayoub — *Preuve de la conservativité pour les motifs en caractéristique nulle*

La conjecture de conservativité affirme que les foncteurs de réalisation classiques des motifs géométriques de Voevodsky sont conservatifs, i.e., détectent les isomorphismes. Il s'agit d'une conjecture centrale de la théorie des motifs ayant des applications concrètes aux cycles algébriques (conjecture de Bloch, etc.). Nous présentons quelques aspects de la preuve de cette conjecture en caractéristique nulle.

Kęstutis Česnavičius — *Purity for the Brauer group*

A purity conjecture due to Grothendieck and Auslander-Goldman predicts that the Brauer group of a regular scheme does not change after removing a closed subscheme of codimension at least 2. The combination of several works of Gabber settles the conjecture except for some cases that concern p-torsion Brauer classes in mixed characteristic $(0, p)$. We will discuss an approach to the mixed characteristic case via the tilting equivalence for perfectoid rings.

Vladimir Chernousov — *Classification of torsors over Laurent polynomial rings*

We will talk about classification of torsors of reductive group schemes over Laurent polynomial rings and applications in infinite dimensional Lie algebras. Joint work with A. Pianzola and P. Gille.

Najmuddin Fakhruddin — *Lifting Galois representations into reductive groups*

A question originating in Serre's modularity conjecture is whether any continuous homomorphism of the absolute Galois group of a number field F into $G(k)$, where G is a split reductive group and k is a finite field, lifts to $G(W(k))$. This was answered affirmatively by Ravi Ramakrishna for $G = GL_2$ (and $F = \mathbb{Q}$) using a purely Galois theoretic method. Ramakrishna's method was extended to GL_n, $n > 2$, by Clozel-Harris-Taylor and to general G by Patrikis. Their results require that the image of the homomorphism be relatively large.

In this talk, based on joint work with Chandrashekhar Khare and Stefan Patrikis, I will explain a refinement of Ramakrishna's method that can be applied to construct (geometric) lifts even when the image is relatively small.

Roman Fedorov — *Counting points on the global nilpotent cone*

Let X be a smooth projective curve over a finite field. Let G be a split reductive group, let ℓ be a line bundle on X. The *global nilpotent cone* is the moduli stack classifying pairs (E, Φ), where E is a principal G-bundle over X, Φ is a nilpotent section of $\operatorname{ad} E \otimes \ell$. Here $\operatorname{ad} E$ stands for the adjoint vector bundle, nilpotence means that for any local trivialization of E and ℓ, Φ takes values in the nilpotent cone of the Lie algebra of G.

The global nilpotent cone is a stack of infinite type, and, in fact, of infinite *volume*, that is, counting points (E, Φ) of this stack with weight $1/\operatorname{Aut}(E, \Phi)$, we get a divergent series. We explain how to stratify the stack by locally closed substacks of finite volume, and address the question of counting points of this stack.

This question for $G = GL(n)$ is an important step in O. Schiffman's counting of Higgs bundles on curves. For groups other than $GL(n)$ the question turns out to be much more complicated.

Michael Harris — *Représentations incorrigibles*

De sa correspondance de Langlands locale numérique, Henniart a déduit le théorème suivant : si F est un corps local non-archimédien, et si π est une représentation irréductible de $\mathrm{GL}(n, F)$, alors, après une suite finie de changements de base cycliques, l'image de π contient un vecteur fixé par un sous-groupe d'Iwahori. Ce résultat a été indispensable dans toutes les démonstrations de la correspondance locale. Scholze en a donné une autre

démonstration, basée sur l'analyse des cycles proches dans la cohomologie de la tour de Lubin-Tate. Le théorème analogue devrait être vrai pour n'importe quel groupe réductif, mais les deux démonstrations connues marchent uniquement pour GL(n). J'esquisserai une troisième démonstration, basée sur les propriétés des fonctions L, qui devrait avoir des applications dans le cadre de la paramétrisation locale de Genestier-Lafforgue.

Javier Fresán — *Hodge theory of Kloosterman connections*

Recently, Broadhurst and Roberts studied the global L-functions associated with symmetric powers of Kloosterman sums and conjectured a functional equation after extensive numerical computations. By the work of Yun, these L-functions correspond to usual motives over Q arising as subquotients of varieties closely related to the geometric Langlands program. For the purpose of computing the Hodge numbers or relating the L-functions to periods, it seems however more convenient to change gears and work with exponential motives. I will construct the relevant motives and show how the irregular Hodge filtration allows one to explain the gamma factors at infinity in the functional equation. If time remains, I will also discuss the conductor. Based on work in progress with Claude Sabbah and Jeng-Daw Yu.

Tasho Kaletha — *Supercuspidal L-packets*

Harish-Chandra has given a simple and explicit classification of the discrete series representations of reductive groups over the real numbers. We will describe a very similar classification that holds for a large proportion of the supercuspidal representations of reductive groups over non-archimedean local fields (which we may call regular). The analogy runs deeper: there is a remarkable parallel between the characters of regular supercuspidal representations and the characters of discrete series representations of real reductive groups. This leads to an explicit construction of the local Langlands correspondence for discrete Langlands parameters with trivial monodromy, under mild conditions on the residual characteristic.

Arthur-César Le Bras — *Overconvergent relative de Rham cohomology over the Fargues-Fontaine curve*

I will explain how to construct a cohomology theory on the category of separated quasi-compact smooth rigid spaces over $\mathbf{C}_p$ taking values in the category of vector bundles on the Fargues-Fontaine curve, which extends Hyodo-Kato cohomology when the rigid space has a semi-stable proper formal model over the ring of integers of a finite extension of $\mathbf{Q}_p$.

George McNinch — *Reductive subgroup schemes of a parahoric group scheme*

Let K be the field of fractions of a complete DVR A with residue field k, and let G be a connected and reductive linear algebraic group over K. Bruhat-Tits associate to G various parahoric group schemes P. Such P are smooth and affine group schemes over A, but in general they are not reductive.

Assume that G splits over an unramified extension of K and that P is an associated parahoric group scheme. In that case, we prove that there is a reductive subgroup scheme M of P such that M_k is a Levi factor of the special fiber P_k, and such that M_K is a reductive subgroup of G containing a maximal torus. In fact, M_K is – at least geometrically – the centralizer of the image of a homomorphism $\mu_N G$ for some $N > 1$.

The talk will describe the construction of M, and it will describe some application of the existence of M to the study of $G(K)$-orbits on nilpotent elements of Lie(G).

Sophie Morel — *TBA*

Igor Rapinchuk — *Algebraic Groups with Good Reduction and Unramified Cohomology*

Let G be an absolutely almost simple algebraic group over a field K, which we assume to be equipped with a natural set V of discrete valuations. In this talk, our focus will be on the K-forms of G that have good reduction at all $v \in V$. When K is the fraction field of a

Dedekind domain, a similar question was considered by G. Harder; the case where $K = Q$ and V is the set of all p-adic places was analyzed in detail by B.H. Gross and B. Conrad. I will discuss several emerging results in the higher-dimensional situation, where K is the function field $k(C)$ of a smooth geometrically irreducible curve C over a number field k, or even an arbitrary finitely generated field. These problems turn out to be closely related to finiteness properties of unramified cohomology, and I will present available results over various classes of fields. I will also highlight some connections with other questions involving the genus of G (i.e., the set of isomorphism classes of K-forms of G having the same isomorphism classes of maximal K-tori as G), Hasse principles, etc. The talk will be based in part on joint work with V. Chernousov and A. Rapinchuk.

Alexander Smith — *TBA*

Nicolas Templier — *On the Ramanujan conjecture for automorphic forms over function fields*

Let G be a reductive group over a function field of large enough characteristic. We prove the temperedness at unramified places of automorphic representations of G, subject to a local assumption at one place, stronger than supercuspidality. Such an assumption is necessary, as was first shown by Saito-Kurokawa and Howe-Piatetskii-Shapiro in the 70's. Our method relies on the l-adic geometry of Bun_G, and on trace formulas. Work with Will Sawin.

Jared Weinstein — *A Lefschetz-Verdier formula for diamonds*

We extend the scheme-theoretic Lefschetz fixed-point formula to an interesting situation, where a profinite group acts on a rigid-analytic space (or a perfectoid space, or a diamond, or a diamond stack). One wants to know the action of this group on tale cohomology. The fixed points locus may well be non-isolated; in fact it could be something like a profinite set. Under a hypothesis of "strong reflexivity", we give a formula for the trace distribution of the group on cohomology, in terms of local terms coming from the fixed point locus. This is joint work with Tasho Kaletha and David Hansen.

Fei Xu — *Arithmetic purity of strong approximation for linear algebraic groups*

Strong approximation property is not birationally invariant. Inspired by simply connectedness property and the example of affine spaces, Colliot-Thelene and Wittenberg asked if strong approximation property is invariant among smooth varieties up to a closed sub-variety of codimension at least 2. In this talk, we will show that this is true for a semi-simple simply connected quasi-split linear algebraic group. We also explain that this phenomenon for strong approximation is linked to Brauer-Manin obstruction. This is a joint work with Yang Cao and Yongqi Liang.